1. (a) We will disprove this by providing a counterexample
Let p = 13
$p + 2 = 15$
$15 = r * s$ where $r = 3$ and $s = 5$
therefore, since neither r nor s equals 1, p is prime, but $p + 2$ is not prime

(b) Let m and n both be even, thus m-n and m+n can be written as
$m - n = 2k - 2l = 2(k - l)$
$m + n = 2k + 2l = 2(k + l)$
thus, $m - n$ and $m + n$ are both even.
Now, let m and n both be odd, thus m-n and m+n can be written as
$m - n = (2k + 1) - (2l + 1) = 2(k - l)$
$m + n = (2k + 1) + (2l + 1) = 2(k + l + 1)$
thus, m+n and m-n are both even.
now, let m be odd and n be even
$m - n = (2k + 1) - 2l = 2(k - l) + 1$
$m + n = (2k + 1) + 2l = 2(k + l) + 1$
thus m-n and m+n are both odd. This proves that for all integers m and n, m-n and m+n are either both odd or both even

(c) We can disprove by providing a counter-example
Let $x = 5.3$ and $y = 4.8$
$\lfloor 5.3 \rfloor - \lfloor 4.8 \rfloor = 5 - 4 = 1$
$\lfloor 5.3 - 4.8 \rfloor = \lfloor 0.5 \rfloor = 0$
thus disproving this claim

(d) If we let x be even, it can be written as $x = 2k$ for some integer k
then, $(2k)^2 - (2k) - 3$
$4k^s - 2k - 3 = 2(2k^2 - k - 2) + 1$
so with x being even, $x^2 - x - 3$ is odd
now let x be odd and written as $x = 2k + 1$
then $(2k + 1)^2 - (2k + 1) - 3$
$= 4k^2 + 4k + 1 - 2k + 1 - 3 = 4k^2 + 2k - 1$
$= 2(k^2 + k - 1) + 1$
thus when $x = 2k + 1$ $x^2 - x - 3$ is odd which proves the statement

(e) let m be any even natural number, we can write it as
$m = 2k$
$m^7 = (2k)^7$
$m^7 = 2^7 * k^7$
$m^7 = 2(2^6 * k^7)$
so $m^7 = 2l$ where $l = (2^6 * m^7)$ thus proving that if m is an even natural number, then m^7 is also even.

2. $d|a \rightarrow d|ax$ because if d divides a, then a is a multiple of d. So for any x, ax is still a multiple of d. This logic can be applied similarly to $d|b \rightarrow d|by$
then, $d|a \wedge d|b \rightarrow d|(a + b)$ because
$d|a \rightarrow dq = a$
$d|b \rightarrow dk = b$
$a + b = dq + dk = d(q + k)$ thus $d|(a + b)$
finally, $d|ax \wedge d|by \rightarrow d|(ax + by)$

3. given x, y, z if $x - y$ is odd, and $y - z$ is even, then they can be written as:
   $x - y = 2k + 1$ for some integer k and $y - z = 2l$ for some integer l
   now, we will isolate x and z to test the proof.
   $x = 2k + y + 1$ and $z = -2l + y$
   then, $x - z = 2k + y + 1 + 2l - y = 2(k + l) + 1$ thus, $x - z = 2n + 1$ where $n = (k + 1)$
   therefore $x - z$ is odd

4. To prove that if r is irrational, then $r^{\frac{1}{t}}$ is also irrational, we will prove the contrapositive.
   Start by assuming that $r^{\frac{1}{t}}$ is rational so $r^{\frac{1}{t}} = z$ such that $z \in \mathbb{Q}$
   since $z \in \mathbb{Q}$, $z = \frac{a}{b}$ such that $b \neq 0$ and $a, b \in \mathbb{Z}$
   $r^{\frac{1}{t}} = z$
   $(r^{\frac{1}{t}})^t = z^t = r$
   $r = (\frac{a}{b})^t = \frac{a^t}{b^t}$ where $t > 0$
   thus $b^t \neq 0$ and $a^t, b^t \in \mathbb{Z}$ because a^t and b^t are just the product of integers
   and therefore r is a rational number which proves the contrapositive