# Propagation of thermal diffusive waves in a metal by Fourier analysis

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**Abstract**

A temperature wave propagates along a long thin bar of a metallic sample when subjected to periodic heating. In this way it is demonstrated that there is no wave nature in these improperly called thermal waves by showing that they do not transport energy and its propagation properties can be used to determine the thermal diffusivity of the material.

## 1 Introduction

A metallic sample heated by a periodic heat source, the resulting temperature oscillations inside the sample have the same mathematical expression as highly damped waves, the so called thermal waves. It must be pointed out that thermal waves cannot be considered as real traveling waves because they show neither wave fronts nor reflection and refraction phenomena so it is demonstrated that there is no wave nature in these improperly called thermal waves because they do not transport energy [1].

The purpose of this experiment is to understand the basis of heat flow, recognize heat conduction as a diffusive process by Fourier analysis, solutions of the heat equation, decompose an oscillation into its harmonics, observe different harmonics and how they damp with different rates, and ultimately calculate the thermal diffusivity of a metal [2].

## 2 Theoretical background

### 2.1 Fourier solution for thermal conduction

The special case relevant to our problem is one-dimensional heat conduction through temporally periodic boundary conditions. Given the periodic nature of the heating function, the solution in the form of a Fourier series:

$$T(x,t) = P_1 x + \Big\langle T(0) \Big\rangle - \frac{4\Delta T}{\pi^2} \sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n^2} e^{-x/d_n} \cos\left(n\omega_1 t - \frac{x}{d_n}\right), \qquad (1)$$

where $d_n = \sqrt{\frac{2D}{\omega_n}}$ are "damping lengths" and $P_1$ is the gradient of the average temperatures. The oscillatory component of the above solution is a periodic function comprising the pulsing frequency $\omega_1$ and only its odd multiples ($\omega_3 = 3\omega_1$, $\omega_5 = 5\omega_1$, $\omega_7 = 7\omega_1$, etc.). The damping lengths $d_n = \sqrt{\frac{2D}{\omega_n}}$ are mathematically similar to the skin depth and represent the distance over which the amplitude of each harmonic decreases to $1/e$ of its value at $x = 0$. As $d_n \propto 1/\sqrt{n}$, the higher harmonics damp away at smaller distances; ultimately, only the fundamental frequency will survive far from the heat source.

# 3 Experimental procedure

Fig. 1 shows the experimental setup. Four K-type thermocouples were clenched equidistantly to a rod of copper of length about 0.5 m and diameter 30 mm. The metallic rod was heated by a square pulse using a 25 W cartridge heater at a rate of 5 mHz. The heater was connected to a switching circuit which was controlled by using a Labview program which sends a square pulse to the relay.

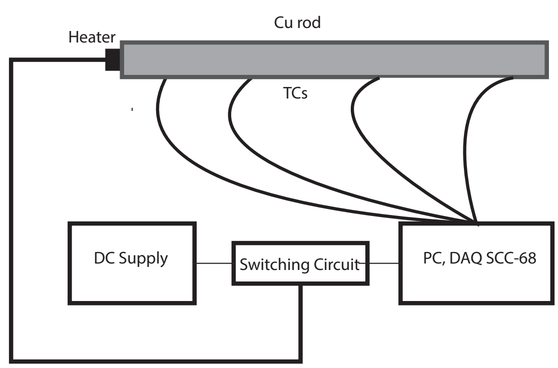


Figure 1: Schematic diagram of the experimental setup.

The thermocouples are first calibrated using Stein-Hart Calibration. DAQ card is attached to collect the data from thermocouple and plot the predicted temperature values as a function of time.The process of heating was continued until the dynamic equilibrium had been achieved after the initiation of the setup.
Once the dynamic equilibrium had been achieved, Fast Fourier Transform (FFT) was performed on the finely sampled numerical data sets and then was plotted. The odd harmonics were seen by FFT graphs. With the help of Fourier Transformed graph, the amplitudes of temperature oscillations (in a dynamic equilibrium) of the first thermocouple (TC1) and the fourth thermocouple (TC4) were measured. With the help of those, the damping coefficient and the velocity of thermal wave

was calculated. Once the damping coefficient and velocity of the thermal wave had been calculated, the thermal diffusivity "$D$" was calculated by $D = v/2\epsilon$. Here "$\epsilon$" represents the damping coefficient.

# 4 Results

Fig. 2 shows that the amplitude of the oscillations decreases with the distance far from the origin. This also illustrates that these oscillations are not in phase; there is a phase lag between successive thermocouples. The triangular shape arises out of the choice of actuation frequency and the distance of the first thermocouple from the heater surface. At first thermocouple, there is not fluctuation in the frequency (maximum amplitude) that is why there occurs triangular variation. At the thermocouple which is farthest from the heat source, the temperature fluctuation (much smaller in amplitude) is nearly a perfect sinusoidal.

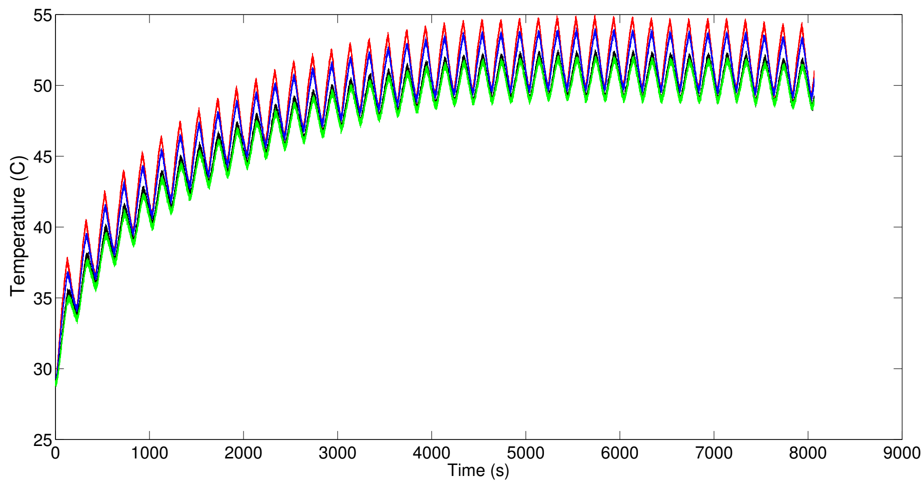


Figure 2: Temperature oscillations at different points along the copper bar. The thermocouples which are nearer to the heat source have higher average temperatures.

Now by looking at the Fig. 3 the thermocouples which are closer to the heat source have the larger amplitudes as compared to the thermocouples which are farther away from the heater. It also shows that there occur only odd harmonics. There are only three odd harmonics which decay exponentially.

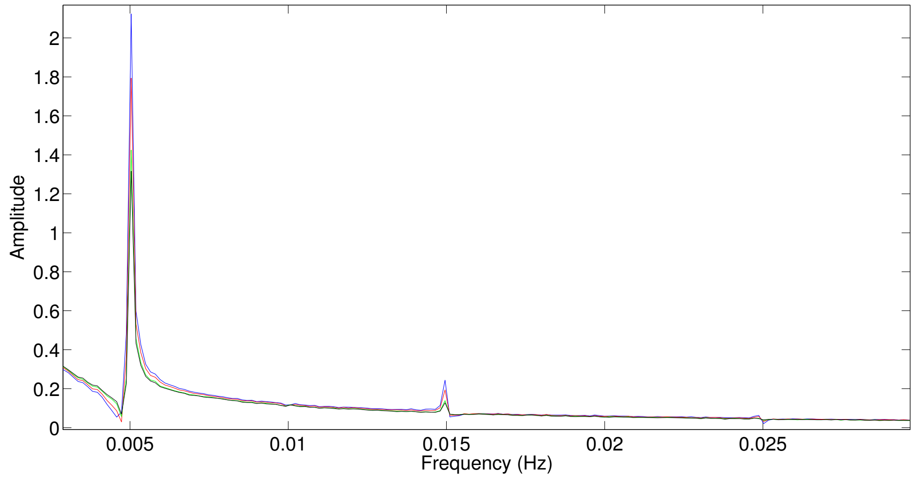


Figure 3: Fourier transforms of the temperatures measured by the thermocouples.

In order to calculate the diffusivity of the material, damping coefficient $\epsilon$ has to be calculated by using following expression

$$\epsilon = \frac{1}{\Delta x} \ln(\frac{A_1}{A_2}) \,, \tag{2}$$

where $A_1$ and $A_2$ are the respective amplitudes of the first and the fourth thermocouple's oscillations and $\Delta x$ represents the separation distance between them which is 0.06 m in this experiment. The values of the amplitudes and the phase lag has been taken from the Fig. 4 and tabulated in the Table 1 and Table 2 respectively.

Table 1: Amplitudes of the oscillations of 1st and 4th thermocouple.

| | Lower amplitude ($L_1$) | Higher amplitude ($L_2$) | Difference ($L_2$-$L_1$) |
|---|---|---|---|
| $A_1$ | 49.81 | 54.77 | 4.96 |
| $A_2$ | 48.93 | 51.92 | 2.99 |

Table 2: Phase lag between 1st and the 4th thermocouple.

| 1st thermocouple phase ($t_1$) | 4th thermocouple phase ($t_2$) | $\Delta t$(s) |
|---|---|---|
| 6034 | 6042 | 8 |

Now from the Eq. 2, the damping coefficient ($\epsilon$) is 9.235 $m^{-1}$. With the help of phase lag ($\Delta t$), the wave velocity has been calculated by $v = \Delta x/\Delta t$ and that is 0.0075 $ms^{-1}$.

Hence, the thermal diffusivity is calculated by following expression

$$D = \frac{v}{2\epsilon} = 4.061 \times 10^{-4}\, \mathrm{m^2 s^{-1}} \tag{3}$$

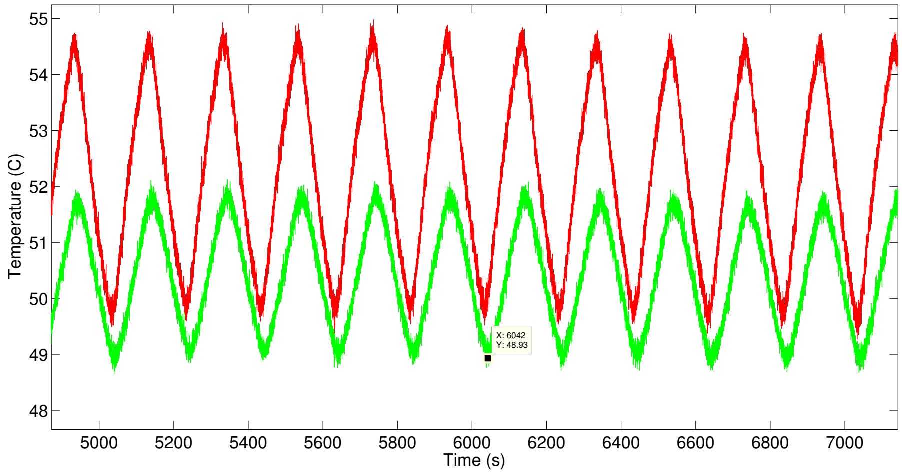


Figure 4: Fourier transforms of the temperatures measured by the thermocouples.

# 5 Conclusion

The purpose of this experiment was to measure the thermal diffusivity of the copper metal. Hence that purpose is achieved and the experimental value is close enough to the theoretical value [4]. This experiment provides an opportunity to get acquainted with heat conduction in a way that is essentially different from that of classical experiments on stationary heat transmission. This experiment also allows one to learn thermal diffusivity measuring techniques in a simple and pedagogical way.

# References

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