The Quadratic Formula

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Abstract

This paper will state and prove the quadratic formula.

1 Introduction

We have all, at some point, learned the quadratic formula. Usually, we learn how to use it to find solutions of a quadratic equation. We might have seen the proof at this point, but most of us did not commit it to memory since the proof does not help us to use the formula.

2 The Quadratic Formula

Theorem 1. *Let a,b,and c be real numbers. The solutions of $ax^2+bx+c=0$ are*

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Proof. In order to prove the quadratic formula, we use the process of completing the square. Starting with $ax^2+bx+c=0$, arithmetic gives

$$ax^2+bx=-c,$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a},$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2},$$

$$(x + \frac{b}{2a})^2 = -\frac{4ac}{4a^2} + \frac{b^2}{4a^2},$$

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b - 4ac}{4a^2}}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$