

# Fundamentals of Physics 3 Homework Template

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## 1 Introduction

You can leave the automatic numbering for sections as is or you can suppress the numbering by using `\section*` instead of `\section`.

### Problem 2.7

Find the electric field a distance  $z$  from the center of a sphere of uniform surface charge  $\sigma$  with radius  $R$ .

**Solution** The surface element of the sphere is

$$da' = (R d\theta')(R \sin \theta' d\phi') \quad (1)$$

$$\mathbf{r} = z \hat{\mathbf{z}}$$

$$\mathbf{r}' = R \hat{\mathbf{r}}$$

$$\mathbf{r} = \mathbf{r} - \mathbf{r}' = z \hat{\mathbf{z}} - R \hat{\mathbf{r}}$$

Let's start by breaking  $\hat{\mathbf{r}}$  into Cartesian coordinates:

$$\hat{\mathbf{r}} = \cos \theta \hat{\mathbf{z}} + \sin \theta (\cos \phi \hat{\mathbf{x}} + \sin \phi \hat{\mathbf{y}}) \quad (2)$$

$$\begin{aligned} \mathbf{r} &= \mathbf{r} - \mathbf{r}' \\ &= z \hat{\mathbf{z}} - R[\cos \theta \hat{\mathbf{z}} + \sin \theta (\cos \phi \hat{\mathbf{x}} + \sin \phi \hat{\mathbf{y}})] \\ &= -R \sin \theta \cos \phi \hat{\mathbf{x}} - R \sin \theta \sin \phi \hat{\mathbf{y}} + (z - R \cos \theta) \hat{\mathbf{z}} \end{aligned} \quad (3)$$

Next, we use the law of cosines to the magnitude of  $\mathbf{r}$ .

$$(4)$$

$$\begin{aligned} \mathbf{E} &= \int \frac{1}{4\pi\epsilon_0} \frac{\hat{\mathbf{r}}}{r^2} \sigma da' \\ &= \frac{\sigma}{4\pi\epsilon_0} \int \frac{\mathbf{r}}{r^3} R^2 \sin \theta d\theta d\phi \end{aligned}$$